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ANALYTICAL MODELS

FOR

SUPPLEMENTING SHIP MANNING SIMULATIONS

by

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ABSTRACT

This report reviews various mathematical models of use in relating ship readiness and system reliability to repair capability. It includes a model that describes the possible effects of preventive maintenance upon readiness. Certain parameters in the models are influenced by the quality of personnel available.

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1. Introduction.

Complex computer simulations, such as that dubbed SHIP II by the Naval Personnel Research and Development Lab, have been developed for the purpose of studying the effectiveness of manpower on board a ship. These simulations attempt to adhere closely to Naval manning doctrine, and are elaborate and detailed, require extensive data inputs, and are expensive to run.

I suggest that such complex models are often usefully paralleled and supplemented by a family of simple analytical or mathematical models, i.e. mathematical structures that, while simplified and abstract, can be manipulated by mathematical techniques. While such models may appear oversimplified or naive, their true function is to provide a broad-brush picture of an admittedly complicated situation and to help explain qualitative phenomena revealed by simulation runs. The existence of a mathematical or probabilistic model that is susceptible to mathematical or simple numerical manipulation also provides one way of checking an elaborate (and accident prone) simulation program. In addition, such a model may serve well as a variance reduction agent; see the account of the "control variables" technique in Gaver [2], and its application in Gaver and Shedler [3].

In this paper I will outline and explore a selection of the kinds of mathematical models that seem especially relevant in the SHIP II context. Further elaborations will be deferred for study in later reports.

2. The "Repairman" Model Type.

The SHIP II model's impact for manpower planning stems from its capacity to relate ship readiness to manning. That is, certain tasks must routinely be required of the crew (evolutions, watch standing, etc.), while others arise in an unpredictable or random manner, as is true of corrective maintenance that is called for when crucial equipments or systems fail. If such failures happen close together in time it may be that waiting will occur for the beginning of repair if few, or inexperienced, repairmen are aboard ship. The reason is that repair backlogs will tend to exist. We wish to analyze the relationship between the supply of repairmen and the readiness or availability of key equipments aboard ship.

Classical models or model types that relate to the above question are the "repairman" models; see Feller [1], and Morse [4]. We outline the usual model assumptions and their most transparent consequences. Then we discuss realistic modifications.

(2.1) Simplest Assumptions.

Suppose that $m (m \geq 1)$ failure-prone equipments are maintained by a group of repairmen. Classically, individual repairmen were authorized to carry out service, but in the real context teams of various constitutions and sizes are required. For the moment suppose that $r (r \geq 1)$ teams are available. Let all equipments have the same basic failure rate, λ, and let μ denote the repair rate (so $\frac{1}{\mu}$ is mean or expected repair time). A single instance of repair time, R, is an exponentially distributed random variable, and the time to failure of an individual equipment is also exponential. All of these

variables are mutually independent. Already we take note of several heroic simplifying assumptions, some of which can be relaxed in a straightforward manner, as we will show. For example, failure times of different equipments will not have the same exponential distribution, nor will repair times. In particular, repair times do not now reflect behavioral factors such as learning and training (neither does the current version of SHIP II). But if a complex simulation, say, allows for arbitrary distributions, then when we specialize to exponentials the simulation should give the same answers as do our models. Thus our models furnish a means of checking internal validity. To the extent that the specific distributional assumptions are irrelevant (for example, results may depend only upon the means of the distributions) then the models furnish formulas that can actually replace time-consuming, expensive, error-prone simulations. This is something to consider and exploit.

Finally, the models assume that repairs are conducted in the arrival order of the failures, without priority assignments. And only the long-run or steady state solutions are found.

(2.2) Model Questions and Answers.

Let $P_n(t)$ denote the probability that exactly n equipments are "down" for repair at time t after, say, a mission or tour commences. Presumably then $P_n(0) = 0$ for $n = 1,2,...,m$, while $P_0(0) = 1$ if the shore repair facilities are successfully preparing the vessel for its tour.

Now Feller ([1], Chap. XVII, Section 7) describes the way in which the latter problem may be formulated as a birth and death process. I will not repeat the mathematical arguments here. In summary the results and their usefulness in present context, are as follows.

(a) Differential equations are derived for $P_n(t)$ in terms of λ, μ, m, and r. The latter are capable of explicit solution in terms of exponentials, but these solutions are complicated. Of more use are numerical solutions, presented graphically. For example, one can tabulate the average (mean, or expected) number of equipments down (unready) and undergoing or awaiting repair at a particular time during a mission as the latter depends upon manning level, r.

(b) It is pointed out that as t, the time that elapses following mission start, becomes large $(t\to\infty)$, then $P_n(t)$ approaches a limit, p_n. This limiting, or steady-state probability distribution can be found nearly explicitly. Actually it is easiest to write down the "probability balance equations"

$$
\begin{aligned}
(n+1)\mu\ p_{n+1} &= (m-n)\lambda\ p_n, \qquad n < r \\
r\ \mu\ p_{n+1} &= (m-n)\lambda\ p_n, \qquad n \geq r
\end{aligned}
\tag{2.1}
$$

and solve them numerically on a computer. That is, we start with $p_n = k_n\ p_0$, $k_0 = 1$, substitute into the above equations and cancel out p_0 and solve successively for $k_1, k_2, \ldots, k_m$. Then, since

$$\sum_{n=0}^{m} p_n = 1 = p_0 \sum_{n=0}^{m} k_n, \tag{2.2}$$

p_0 may be determined.

These "steady state" solutions can be used to calculate an approximation to the expected number of equipments down (unready) after the mission has been under way for some time. Feller [1] shows that the expected number of machines in line, waiting to begin processing or repair, is

$$w = m - \left(\frac{\lambda+\mu}{\lambda}\right)(1-p_0) \tag{2.3}$$

Even if we don't know p_0, we do know that it lies between zero and unity, so very crudely

$$m - \left(\frac{\lambda+\mu}{\lambda}\right) \le w \le m, \tag{2.4}$$

and better approximations can be derived with some effort. Work is currently underway to build useful mathematical approximations to such problems based on simplifications that occur when m becomes large. But much that is useful as a supplement to (or replacement of) simulations follows from adaptations of the above results.

(2.3) <u>Alternative Assumptions and Results</u>: <u>The Infinite Server Approximation</u>.

The assumptions in the previous section can, with benefit, be relaxed in one direction, and made more general in another, if a different model is constructed.

I will again assume that equipments break down at random, i.e. the i^{th} equipment has failure rate λ_i $(i = 1,2,3,\ldots,m)$. The

repair time distribution of the $i^{\underline{th}}$ equipment is $F_i(x)$, where

$$F_i(x) = P\{R_i \leq x\} \tag{2.5}$$

is a completely arbitrary distribution, e.g. the log-normal. This is a more general assumption than was made earlier, for there R_i was exponential. Furthermore, assume that the repair process is such that there is <u>no waiting to begin repair</u> because of the presence of other, prior failing, equipments. This rather strong assumption can perhaps be justified if equipments do indeed fail infrequently enough, and repair times are short enough. It will <u>not</u> be justified if there are only a few repairmen (or teams), as would be the case if there were a drastic reduction in force, or if the quality of repair service degenerated, i.e. repair times increased because of inexperienced crews. Results that may be obtained (we omit details) are as follows.

(a) The <u>long-run</u> or <u>steady-state</u> probability that a given system, System i, is undergoing repair is

$$P\{\text{System } i \text{ unavailable}\} = \frac{\lambda_i E[R_i]}{1 + \lambda_i E[R_i]} \tag{2.6}$$

Actually, one can find the probability that System i is, or is not, available at <u>any</u> time t following mission start with somewhat more difficulty.

<u>Note</u>. The above formula, (2.6), holds regardless of whether failure times are exponentially distributed or not. It turns out that <u>long-run</u> availability depends only upon mean or expected

time to failure, λ_i^{-1}, in this model. Consequently, even if times to failure have the Weibull, log-normal, or gamma distributions, our formula holds well, and can be used to check out or validate simulations.

(b) As a result of (a), the expected number of systems that are unavailable is

$$E[\# \text{ of Systems unavailable}] = \sum_{i=1}^{I} \frac{\lambda_i E[R_i]}{1 + \lambda_i E[R_i]},$$

and the variance of the number of unavailable equipments is

$$Var[\# \text{ of Systems unavailable}] = \sum_{i=1}^{I} \frac{\lambda_i E[R_i]}{(1+\lambda_i E[R_i])^2}$$

(c) Furthermore, the probability that all systems are available is

$$P\{\text{all Systems available}\} = \prod_{i=1}^{I} \left(\frac{1}{1+\lambda_i E[R_i]}\right)$$

because systems are supposed to fail and be repaired entirely independently. This need not be a good model, but can be used to check internal validity.

(d) Finally, if all $\lambda_i E[R_i]$ terms are of about the same magnitude and are small compared to unity, as should frequently be the case, then the distribution of the total number of Systems out or unavailable will be approximately Poisson distributed.

Note. The above formulas are simple, explicit, and hence easy to apply. They should approximate actual system availability late in a mission if the failure rate is reasonably small; the restriction to "late in a mission" is necessary because our results are for the steady state, which requires time to establish itself. However, our steady-state results (average number of systems down, for example) will typically be more optimistic than they should be, because of the assumption that a repair may always begin immediately. Thus our simple models can be immediately employed to generate an upper bound on availability or readiness.

3. More Realistic Repairman Models.

Certain simplifying assumptions made in the models just described may be rectified with only a little trouble. The changes we suggest will make the results more credible and valid, but, as seems inevitable, there will be costs. The costs present themselves in the form of increased mathematical complexity in model formulation and manipulation. Nevertheless, these particular difficulties are quite surmountable if a digital computer is available to carry out computations. Other costs relate to the development of a better understanding of the maintenance process, the gathering of relevant data on the effectiveness of training, etc. Such data might come from fleet experiments and service school experience. At the moment our models can only answer interesting "what if" questions, consuming judgmental data.

(3.1) Individualizing the Repairman Models.

The basic repairman models assume that failure rates (demand for repairs) of equipments are equal. In fact, this is rather unlikely. Let us assume instead that there are m equipments with failure rates λ_i $(i = 1,2,\ldots,m)$, and one repairman (or crew) that services them all. This latter restriction may easily be removed, and we make it only to illustrate in a simple way the new analysis, which is straightforward but tedious and is not in the standard books. We will assume that each equipment has its own repair rate (repair times are exponentially distributed): repair rate for system i is μ_i $(i = 1,2,\ldots,m)$.

Because the failure rates differ it is necessary to specify a descriptive set of states for the entire system. I will first illustrate

this for only two systems or machines (e.g. a sonar and a computer), leaving it up to the reader to generalize.

The Two-Equipments, One Repair Group, System.

Consider the following possible states in which the system may find itself. Of course, the states may be identified in any way, e.g. by letters; the numbers we use mean nothing intrinsically.

Equipment 1	Equipment 2	State
Up	Up	0
Up	Down	1
Down	Up	2
Down First	Down Second	3
Down Second	Down First	4

Note especially the last two lines of the table: if repair is conducted on a first-come, first-served basis the noted distinction must be made. To explain, consider State 4: if this state prevails, Equipment 1 failed first (and is undergoing repair), while Equipment 2 is also down, but is awaiting repair. The reverse is true in State 3. If priority repair is carried out, with priority determined by expected repair time duration, then a simpler table (and fewer equations) may result.

It is straightforward to write down the descriptive differential equations. We make the usual assumptions. Let $P_j(t)$ be the probability that the system is in state j. We have, up to terms negligible compared to dt,

$$P_0(t+dt) = P_0(t)[1-(\lambda_1+\lambda_2)dt] + P_1(t)\mu_2 dt + P_2(t)\mu_1 dt \qquad (3.1)$$

so, subtracting $P_0(t)$ from both sides and taking limits yields

$$\frac{dP_0}{dt} = -(\lambda_1+\lambda_2)P_0 + \mu_2 P_1 + \mu_1 P_2 \qquad (3.2)$$

the steady-state equation is

$$(\lambda_1+\lambda_2)p_0 = \mu_2 p_1 + \mu_1 p_2 \qquad (3.3)$$

By a similar argument we write down

$$(\lambda_1+\lambda_2)p_1 = \lambda_2 p_0 + \mu_1 p_3 \qquad (3.4)$$

$$(\mu_1+\lambda_2)p_2 = \lambda_1 p_0 + \mu_2 p_4 \qquad (3.5)$$

$$\mu_1 p_3 = \lambda_2 p_2 \qquad (3.6)$$

$$\mu_2 p_4 = \lambda_1 p_1 \qquad (3.7)$$

Furthermore,

$$p_0 + p_1 + p_2 + p_3 + p_4 = 1 \qquad (3.8)$$

since exactly one state must prevail at any time. Thus, in order to find the probabilities we must leave out one equation among (3.3)-(3.7), but retain (3.8). In truth, these equations may be solved simultaneously to yield an algebraic formula in this simple case, but it will probably be more effective to carry out solutions numerically after plugging in suitable parameter values. A digital computer will do this problem quite easily, utilizing standard codes for solving linear

equations. In general, the number of equations will be about 2^m, although this represents a low estimate in the present case. It is far easier to count the number of machines in each condition (up, down, undergoing repair, or waiting) than it is to keep track of each machine or equipment. This is why the models described earlier appear in text books, and why the later models, while more realistic, are ignored.

Note that matters strictly related to manpower quality and training specifically enter these models through the repair rate parameters μ_i: the higher the level of skill the larger will be μ_i, and the greater the fraction of time that the equipment will be available. Incidentally, it might be noted that while the elaborate system of states presented is required for analysis, only a few states are really of interest: p_0 is the probability that both equipments are entirely available, p_1 is the probability that Equipment 1 is available and Equipment 2 is not, while p_2 is the reverse (Equipment 2 up, Equipment 1 down).

It should be clear that equations of the same form can be written to describe more complex set ups (more machines, and more teams). Again, although our model is simple it will at least be useful for checking the internal validity of the SHIP II simulation program. Of course, the solutions obtained are not time-dependent: we would anticipate higher readiness towards the start of a cruise or mission than at the end just because all systems should be initially operative. At another time we will discuss time-dependent or transient solutions. These allow

us to trace changes in readiness as a mission progresses; the latter changes may well reflect a repair capability that is inadequate to keep up with failures.

4. Preventive Maintenance and Repair.

Widely held conventional wisdom teaches that preventive maintenance (p.m.) can eliminate or postpone chance failures that may occur during an operation. Thus preventive maintenance may improve readiness, and is included as part of the normal workload for that reason. Several comments may be made, however.

(a) Some preventive maintenance activities may actually reduce readiness, for example, if carried out by inexperienced and unmotivated personnel. Certainly equipment is "down" while testing and maintenance is under way, and logistics problems may be created if unnecessary replacements are made. A service school (for technicians) should provide a good environment for deciding about the efficiency of preventive maintenance involving various equipment types and differing levels of technician competence. An experimental program could be designed for dealing with just this question.

(b) The present SHIP II model recognizes the existence of p.m. as part of workload, but does not relate the effort, or its skill level, to altered time between failures. While an exact relationship would be difficult to establish, a reasonable class of models can be constructed that reflect the behavior desired, and that allows a decision maker to answer "what if?" questions concerning reliability and maintenance activities.

I believe that the formulation of such an explicit model will stimulate further inquiry into the interrelationship between p.m. and

readiness. I have no belief that the simple models suggested here are precisely valid, suggesting only that they are more representative of "real life" than is a model that ignores any possible relationship.

4.1 <u>One System, One Repair Team</u>.

In order to make a beginning, we imagine that one system (a sonar, for example) is served by one repair team. The latter is responsible for p.m. and corrective maintenance (c.m.). We suppose that p.m. is intended to assure that c.m. incidents occur as rarely as possible.

<u>Model A</u>. Let λ_H be a high failure rate (short MTBF), λ_L be a low failure rate (long MTBF), (so $\lambda_L < \lambda_H$), μ be the repair rate, and ν be the rate at which p.m. activities are conducted. The significance of the high failure rate <u>vs</u>. low failure rate (λ_H <u>vs</u>. λ_L) contrast is that presumably the low failure rate occurs if correct, successful, p.m. is performed, while otherwise λ_H is in effect. We also introduce a probability structure for the <u>adequacy of p.m</u>. If current system state is H (so that λ_H prevails) and if a p.m. is performed then we let h $(0 \le h \le 1)$ be the probability that the system's state remains H, while with probability $\bar{h} = 1 - h$ it is inadvertently shifted to L. Similarly, if the system is in L, let ℓ be the chance of remaining in L, while $\bar{\ell} = 1 - \ell$ is the chance of passing to H when p.m. is performed. Thus, "good" p.m. is characterized by h near unity, and ℓ near zero. A little reflection indicates that if the opposite is true then it may be desirable to have a <u>small</u> value for ν (p.m. rate or the rate of introducing p.m. activities)--thus meaning that the time between detrimental tinkerings is long.

We also introduce another set of parameters analogous to h and ℓ in order to describe the results of c.m. For the moment we shall assume that when a failure occurs it is repaired at rate μ, regardless of its failure state just before failure. Also, we will assume that it is restored to the low failure rate (L) state by c.m. with probability π_L, and to the high rate with probability π_H. These probabilities are independent of the past. They reflect the quality of repair service, and hence of the assigned personnel, as is the case with h and ℓ.

The values of ℓ and h, and of π_L and π_H, may differ because of differing personnel policies. For instance, $\bar{h}$ and may be low, yet π_H is high, simply because new or relatively ill-trained personnel are assigned to preventive maintenance, but when a failure (c.m. incident) occurs the better people are brought into the ball game. Whether this is a wise strategy obviously depends upon the relationship between the various parameters; many times it will not be.

Now let $u_H(t)$ be the probability that our system is up at t, and has failure rate λ_H (i.e. is in state H), let $u_L(t)$ denote the probability that the system is up but is endowed with λ_L (is in state L), and let $r(t)$ denote the chance that the system is down for c.m. repair. All of our simple assumptions--many of which can be immediately relaxed--indicate that we are dealing with a simple Markov process in 3 states. Thus, we can write down differential equations for $u_H(t)$, $u_L(t)$, and $r(t)$. Alternative methods of analysis are also useful, especially when exponential distributions are jettisoned. Such methods will be described later.

To derive the differential equations, and then the long-run behavior, arguments go as follows. In order for the system to be in state H at $t + dt$ either (i) it was in H at t and did not change in $(t,t+dt)$, (ii) it was in L at t, preventive maintenance occurred (instantaneously, by present assumption), shifting the system to H, (iii) it was on repair (c.m.) at t, and repair was completed in $(t,t+dt)$; the repair led to the H state. Other events are negligible. Thus

$$u_H(t+dt) = u_H(t)[1-\nu\bar{h}dt-\lambda_H dt] + u_L(t)\nu\bar{\ell}dt + r(t)\mu\pi_H dt,$$

leading to

$$\frac{du_H}{dt} = -[\nu\bar{h}+\lambda_H]U_H + \nu\bar{\ell}u_L + \mu\ell_H r(t). \tag{4.1}$$

Next, a similar argument leads to the equation

$$u_L(t+dt) = u_L(t)[1-\nu\bar{\ell}dt-\lambda_L dt] + u_H(t)\nu\bar{h}dt + r(t)\mu\pi_L dt;$$

leading to

$$\frac{du_L}{dt} = -[\nu\bar{\ell}+\lambda_L]u_L + \nu\bar{h}u_H + r\mu\pi_L \tag{4.2}$$

Finally,

$$u_H(t) + u_L(t) + r(t) = 1, \tag{4.3}$$

and one can solve (4.1), (4.2), and (4.3) simultaneously. To obtain a time dependent solution Laplace transforms may be used; the stationary solution is obtained by setting the derivatives equal to zero in (4.1)

and (4.2) and solving the resulting linear equations. Explicit solutions will be given and discussed later.

Model B. It is helpful to generalize the previous model, and to treat it in an alternative and more general way. To do so we will first write expressions for the distributions of the time to failure (c.m. instants) starting from a repair completion time.

Let

A_L = an up-period duration, beginning with the equipment in state L, having just ended c.m. (or p.m.);

A_H = same, but referring to a c.m. (or p.m.) that initializes the equipment in state H.

Decomposition at the first event, followed by application of the convolution property of the transforms, gives

$$\hat{\alpha}_L(s) \equiv E[e^{-sA_L}] = \int_0^\infty e^{-sx}[1-G(x)]e^{-\lambda_L x}\lambda_L dx$$

$$+ \hat{\alpha}_L(s)\ell \int_0^\infty e^{-sx} e^{-\lambda_L x} G\{dx\} + \hat{\alpha}_H(s)\bar{\ell} \int_0^\infty e^{-\lambda_L x} G\{dx\}; \tag{4.4}$$

and

$$\hat{\alpha}_H(s) \equiv E[e^{-sA_H}] = \int_0^\infty e^{-sx}[1-G(x)]e^{-\lambda_H x}\lambda_H dx$$

$$+ \hat{\alpha}_H(s)h \int_0^\infty e^{-sx} e^{-\lambda_H x} G\{dx\} + \hat{\alpha}_L(s)\bar{h} \int_0^\infty e^{-sx} e^{-\lambda_H(x)} G\{dx\}. \tag{4.5}$$

Here G is the distribution of the time between p.m. moments, measured from a c.m. termination. Simplification of (4.4) and (4.5) yields two simultaneous linear equations for the transforms:

$$\hat{\alpha}_L(s) = \lambda_L\left[\frac{1-\hat{G}(s+\lambda_L)}{s+\lambda_L}\right] + \ell\,\hat{G}(s+\lambda_L)\hat{\alpha}_L(s) + \bar{\ell}\,\hat{G}(s+\lambda_L)\hat{\alpha}_H(s) \quad (4.6)$$

$$\hat{\alpha}_H(s) = \lambda_H\left[\frac{1-\hat{G}(s+\lambda_H)}{s+\lambda_H}\right] + h\,\hat{G}(s+\lambda_H)\hat{\alpha}_H(s) + \bar{h}\,\hat{G}(s+\lambda_H)\hat{\alpha}_L(s) \quad (4.7)$$

where $\hat{G}(s)$ represents the Laplace-Stieltjes transform of the time between successive p.m. moments. We give two examples:

<u>Example 1</u>: $G(x) = 1 - e^{-\nu x}$; $\hat{G}(s) = \nu(\nu+s)^{-1}$. This is the case of random occasions for p.m. on the particular equipment. The expected or average time between inspection is ν^{-1}. Such an assumption may be reasonable if waits occur for crucial personnel who are otherwise occupied doing c.m. or other tasks. It is probably less realistic than the next.

<u>Example 2</u>. $G(x) = \begin{cases} 0, & x < \frac{1}{\nu} \\ 1, & x \geq \frac{1}{\nu} \end{cases}$; $G(s) = e^{-s/\nu}$. This is the case of <u>regular</u> inspections and p.m. actions, at time intervals of exact, non-random length ν^{-1}.

Other distributions for inspection intervals are possible. One reasonable approach would be to determine $G(x)$ empirically, either from actual shipboard data or from simulations or more extensive models.

In order to derive the stationary or long-run probability of readiness we must have the expected values of A_H and A_L, and these

may be derived by differentiating (4.6) and (4.7), and then solving the resulting linear equations. Laplace transform theory tells us that an equivalent result is obtained by computing the limits $\lim_{s\to 0} \frac{1-\hat{\alpha}(s)}{s} = E[A_L]$ and $\lim_{s\to 0} \frac{1-\hat{\alpha}_H(s)}{s} = E[A_H]$. If we work directly with (4.6) and (4.7) we find the equations

$$-\bar{\ell}\ \hat{G}(\lambda_L)E[A_H] + \{1 - \ell\ \hat{G}(\lambda_L)\}E[A_L] = \frac{1-\hat{G}(\lambda_L)}{\lambda_L} \tag{4.8}$$

$$\{1 - h\ \hat{G}(\lambda_H)\}E[A_H] - \bar{h}\ \hat{G}(\lambda_H)E[A_L] = \frac{1-\hat{G}(\lambda_H)}{\lambda_H} \tag{4.9}$$

The solution to this set is

$$E[A_H] = \frac{\left[\frac{1-\hat{G}(\lambda_H)}{\lambda_H}\right][1-\ell\ \hat{G}(\lambda_L)] + \left[\frac{1-\hat{G}(\lambda_L)}{\lambda_L}\right]\bar{h}\ \hat{G}(\lambda_H)}{[1-h\ \hat{G}(\lambda_H)][1-\ \hat{G}(\lambda_L)] - \bar{\ell}\ \bar{h}\ \hat{G}(\lambda_L)\hat{G}(\lambda_N)} \tag{4.10}$$

$$E[A_L] = \frac{\left[\frac{1-\hat{G}(\lambda_L)}{\lambda_L}\right][1-h\ \hat{G}(\lambda_H)] + \left[\frac{1-\hat{G}(\lambda_H)}{\lambda_H}\right]\bar{\ell}\ \hat{G}(\lambda_L)}{[1-h\ \hat{G}(\lambda_H)][1-\ell\ \hat{G}(\lambda_L)] - \bar{\ell}\ \bar{h}\ \hat{G}(\lambda_L)\hat{G}(\lambda_H)} \tag{4.11}$$

From these we can find a general expression for long-run reliability, r, of the system (probability that the system is in the up state).

$$r = \frac{\pi_H\ E[A_H] + \pi_L\ E[A_L]}{\frac{1}{\mu} + \pi_H\ E[A_H] + \pi_L\ E[A_L]} \tag{4.12}$$

Optimum Inspection Interval

It is intuitively clear that if π_L is close to one (so that c.m. usually returns the system to the low failure rate state), while p.m. often switches the system from state L to state H, and if λ_H

is much greater than λ_L, then it is best to avoid p.m. In other words, there may be an optimum p.m. rate, ν_{opt}, that maximizes

$$E[A] = \pi_H \, E[A_H] + \pi_L \, E[A_L], \tag{4.13}$$

and hence also maximizes the reliability r. For any given set of parameter values we can of course explore $E[A]$ as a function of ν by direct computation. Very probably this will be the only feasible way of proceeding, especially when regular inspections and p.m. activities are carried out; see Example 2. In the case of Example 1 we can actually find the optimum interval analytically or in closed form. By simplification of (4.13) with the assistance of (4.10) and (4.11) we find that

$$E[A] = \frac{\pi_H \, \lambda_L + \pi_L \, \lambda_H + \nu(\bar{\ell}+\bar{h})}{(\lambda_H+\nu\bar{h})(\lambda_L+\nu\bar{\ell}) - \bar{\ell} \, \bar{h} \, \nu^2} \tag{4.14}$$

Next, differentiation with respect to ν is carried out, and the result set equal to zero; the obvious concavity shows that at most one interior maximum exists:

$$\nu_{opt}^{-1} = \text{Optimum c.m. Interval} =$$

$$\frac{[(\ell+\bar{h})(\lambda_L \, \bar{h}+\lambda_H \, \bar{\ell}) - \bar{h} \, \lambda_L - \bar{\ell} \, \lambda_H]}{\lambda_H \, \lambda_L - (\bar{h} \, \lambda_L+\bar{\ell} \, \lambda_H)[\lambda_L + \pi_L(\lambda_H-\lambda_L)]} \tag{4.15}$$

One qualitative fact that emerges is that if π_L is increased, the optimum p.m. interval also increases. The effect is to postpone c.m. actions that may cause a change to the high failure rate state. We must, of course, check $E[A]$ evaluated at ν_{opt} with its value when

$\nu = 0$ or $\nu = \infty$, for the true optimum may occur at one of those points for certain parameter values.

5. Concluding Discussion.

I have attempted to present in this report several methods and models that are relevant to studying manning problems of the type addressed by the SHIP II simulation. Although the models here are not as elaborate as those encompassed by SHIP II, more detail can be built in by taking a mathematical approach. We believe that such supplementary modeling should be carried out in parallel with most complex simulations, if only to check on internal validity. Very possibly a simple analytical model can be "fitted" to simulation data, leaving certain parameters free for variation. An example might be the p.m. rate appearing exogenously in our last model: the latter will depend upon manpower level and other tasks, but could be estimated from simulation data with the above parameters held fixed. We can then use the model to estimate the effect of p.m. on reliability without actually sampling. This would result in an obvious and welcome economy in computational time. Avoidance of simulation when possible also avoids the confusion of random error.

Future work in the present area will include consideration of transient behavior of our repair processes. Under reduced, or less skilled, manning, one would anticipate degradation of reliability throughout a mission. This effect can be traced both by simulation and by mathematical analysis. Finally, we plan to consider the presentation of the mathematical results in the form of an interactive computer program that a user can manipulate from a console. This enables an analyst to change parameter values at will and by experimentation build up a sound and useful feeling for the effects of various system parameters.

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13. ABSTRACT

This report reviews various mathematical models of use in relating ship readiness and system reliability to repair capability. It includes a model that describes the possible effects of preventive maintenance upon readiness. Certain parameters in the models are influenced by the quality of personnel available.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability						
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Readiness						
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